Technical Comments

Comment on "Interpolation Using Surface Splines"

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THE solution of the multiply-supported infinite plate problem by Harder and Desmarais[1] is an ingenious resolution of a classical problem in aeroelasticity. However, the authors did not comment on the historical significance of their contribution and were content to demonstrate the superiority of the surface spline over the least accurate of the various alternatives that have been proposed over the years. We wish to review the development of two-dimensional interpolation methods in aeroelasticity and to demonstrate the accuracy of the surface spline by comparison to a more reasonable alternative.

An essential aspect of modern aeroelastic analysis is the aerodynamic/structural interface. As Hitch noted in his 1963 Wright Brothers lecture to the Royal Aeronautical Society[2]: "A corollary to the use of both a lifting surface aerodynamic method and a detailed structural analysis method is that the grids used will not coincide. There is therefore a need to accept values in one grid and by interpolation/extrapolation to obtain values in the other grid. This is no easy matter and strikes right at the heart of the flutter analysis process." The history of interpolation methods in aeroelasticity has been a meandering between accurate methods and convenient methods. The use of station functions was suggested in 1949 by Rauscher,[3] who conceived the idea of station functions as a refinement of the lumped mass type of analysis, taking the continuous mass distribution into account but retaining the simplicity of a finite set of ordinates as the generalized coordinates. Schmitt[4] observed that the station function method "leaves much to be desired in terms of routine machine adaptation" and presented a matrix interpolation formula based on a weighted least-squares curve-fitting procedure. It was Schmitt's experience that "product combinations of the normal modes of a uniform cantilever beam provide very satisfactory functions for plate like wing surfaces." For weighting factors, Schmitt suggested the planform areas of the regions surrounding the structural control points. In 1959, Rodden[5] generalized the technique of Schmitt in order to eliminate the dependence on the least-squares interpolation method and suggested "interpolation-in-the-small in which each small region is fitted with functions appropriate to the significant boundary conditions and structural discontinuities in the region." This appeared to be a more reliable choice than interpolation-in-the-large (in which the entire surface is fitted with a single set of functions) since, "although a practical surface is uniform for the most part, there are always a sufficient number of structural discontinuities to be bothersome." The recommendation for interpolation-in-the-small was a consequence of attempts by Rodden and Revell[6,7] to employ slender-wing theory in aeroelastic analyses of the North American B-70 Bomber. The mathematical elegance of slender-wing theory is such that, within the aerodynamic approximations made, an exact solution for the unsteady loading results if all modes of motion can be expressed as two-dimensional polynomials in the streamwise and chordwise coordinates. Unfortunately, the polynomial surface fits, first in the Lagrangian sense and later in the least-squares sense, were found to be inaccurate, and slender-wing theory was eventually abandoned in design analyses of the aircraft. Still, the convenience of the least-squares polynomial persists,§ and 21-term two-dimensional polynomials have become the basis of many recent unsteady aerodynamic developments, e.g., Ref. 8, in the subsonic, transonic, and supersonic Mach number regimes.

Table 1 SIC's from orthogonal modes and interpolation-in-the-small (in./lb × 10^6)

Load station[a]	Deflection station[a] 4	6	8	11	13	15	17	21	23	25	32	34	42
4	38												
6	89	257											
8	95	388	798										
11	−1	−1	5	13									
13	16	35	38	14	22				(Symmetric)				
15	49	125	167	9	36	97							
17	69	206	338	6	42	142	233						
21	4	10	16	19	22	19	15	28					
23	14	34	46	22	31	44	51	33	45				
25	30	76	114	20	41	86	120	31	56	96			
32	7	19	28	13	18	22	24	20	26	30	16		
34	11	26	38	18	26	37	44	28	38	49	22	33	
42	1	1	2	1	2	3	4	1	2	4	1	2	2

[a] See Refs. 9 or 10, Fig. 4 for station locations.

Received July 21, 1972.
Index category: Structural Static Analysis.

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§ We are reminded of the man looking for a lost coin beneath a street lamp—not because he had lost it there, but because the light was better.

Table 2 SIC's from orthogonal modes and surface-spline interpolation (in./lb $\times 10^6$)

Load station	Deflection station												
	4	6	8	11	13	15	17	21	23	25	32	34	42
4	40												
6	90	251											
8	97	382	786										
11	−1	−2	4	12									
13	15	34	40	13	22			(Symmetric)					
15	50	124	169	9	35	96							
17	70	204	340	6	41	141	232						
21	5	11	17	18	21	19	16	27					
23	15	34	47	21	30	43	49	32	43				
25	30	77	116	20	40	85	120	31	55	95			
32	8	19	28	13	18	22	25	20	26	30	16		
34	11	26	38	18	26	37	44	27	38	49	22	33	
42	1	2	2	1	2	3	4	1	2	4	1	2	0

Table 3 SIC's from symmetrical static deflection test (in./lb $\times 10^6$)

Load station	Deflection station												
	4	6	8	11	13	15	17	21	23	25	32	34	42
4	59												
6	90	254											
8	119	379	875										
11	5	9	12	28									
13	21	38	52	19	34			(Symmetric)					
15	52	119	178	18	41	107							
17	72	203	348	20	50	144	265						
21	7	14	20	27	27	29	32	50					
23	14	30	44	26	35	47	57	43	58				
25	31	74	117	27	46	92	128	43	65	129			
32	5	11	17	20	23	27	31	36	39	43	42		
34	9	21	32	22	30	43	52	40	52	70	45	73	
42	0	1	1	6	7	9	10	12	14	14	21	23	27

It therefore comes as no great surprise that Harder and Desmarais found the surface spline clearly superior to the 21-term least-squares polynomials. A better evaluation of the surface spline can be made by an application to a practical wing structure. Such an example is the built-up 45° delta-wing specimen investigated experimentally in Ref. 9. The calculation of the measured structural influence coefficients (SIC's) from the measured vibration data was attempted in Refs. 10 and 11 and required three interpolation matrices: one relating the mass points to the modal points, another relating the modal points to the SIC points, and the third relating the modal points to the constraint points for the SIC's. The interpolation and constraint matrices were derived in Ref. 10 by interpolation-in-the-small, and the same matrices were utilized in Ref. 11 to orthogonalize the measured modes and to derive the SIC's. The SIC's calculated in Ref. 11 for a representative set of points are shown in Table 1. Repeating the calculations of Ref. 11 using the interpolation and constraint matrices derived by the surface-spline method of Harder and Desmarais leads to the representative SIC's in Table 2. Table 3 presents the experimental SIC's measured in the static test. Comparison of Tables 1 and 2 with Table 3 shows again the large truncation error in SIC's derived from only four vibration modes, but for the purposes of this Comment the comparison of Table 1 with Table 2 shows the surface spline interpolation to be practically equivalent to interpolation-in-the-small. The elements of Table 1 are generally slightly larger than the elements of Table 2 (e.g., the tip SIC, point 8, is 0.000798 in./lb in Table 1 and 0.000786 in./lb in Table 2) which shows, perhaps, that carefully chosen local interpolation functions will always be more accurate than some routine procedure, but the surface spline interpolation certainly has all the accuracy normally expected of an engineering method, and clearly all of the convenience required for routine computation.

References

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THE authors welcome the comments, and the evaluation of the surface spline methods. There are two points which were not sufficiently emphasized in the comparison of surface splines and "interpolation-in-the-small." 1) The success of interpolation-in-the-small requires much work by the analyst to determine the choice of structural points and interpolation functions to be used for each interpolated point. The surface spline technique is more easily automated. 2) The example chosen to compare the two methods is incapable of giving an accurate evaluation. The errors associated with modal truncation are much greater than errors resulting from interpolation, thus completely overshadowing the differences.

Received August 21, 1972.
Index category: Structural Static Analysis.

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